

Strongly Coupled Quark Gluon Plasma (SCQGP)

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Abstract

We propose that the reason for the non-ideal behavior seen in lattice simulation of quark gluon plasma (QGP) and relativistic heavy ion collisions (URHICs) experiments is that the QGP near T_c and above is strongly coupled plasma (SCP), i.e., strongly coupled quark gluon plasma (SCQGP). It is remarkable that the widely used equation of state (EoS) of SCP in QED (quantum electrodynamics) very nicely fits lattice results on all QGP systems, with proper modifications to include color degrees of freedom and running coupling constant. Results on pressure in pure gauge, 2-flavors and 3-flavors QGP are all can be explained by treating QGP as SCQGP as demonstrated here. Energy density and speed of sound are also presented for all 3 systems. We further extend the model to systems with finite quark mass and a reasonably good fit to lattice results are obtained for (2+1)-flavors and 4-flavors QGP. Hence it is the first unified model, namely SCQGP, to explain the non-ideal QGP seen in lattice simulations with just two system dependent parameters.

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1 Introduction :

It is now believed that hadrons are confined state of quarks. Fundamental theory to explain hadrons is quantum chromodynamics (QCD). Non-Abelian nature of QCD leads to complicated non-perturbative structure of QCD vacuum. Non-perturbative QCD vacuum leads to confinement of quarks at low energy. QCD on lattice confirms this and at the same time predicts that, at high energy density or baryon density, hadron goes to deconfined state called QGP [1]. For the last 20 years, lot of experimental, theoretical and lattice simulation of QCD results were out, but there were no conclusive evidence for QGP and order of phase transition. This is probably because the exact nature of matter found in the URHICs or in lattice simulations near T_c is not well understood [2]. Both experimental results [3] and lattice results [4] show that matter formed near T_c is non-ideal even at $T > T_c$ or above [2]. There are lot of attempts to explain such a matter using various models like bag model, other confinement models, quasi-particle models, strongly interacting quark gluon plasma (sQGP) etc.

In bag model [5], QGP is considered as a big hadron with large number of partons interacting weakly confined by bag wall. This model is only partially successful. Inclusions of glue balls or hadrons improve the results. The confinement models are the extension of bag model with smooth potential like Cornell potential [6], relativistic harmonic oscillator [7] etc. Again only partially successful. In quasi-particle models [8] a new concept of calculating thermodynamics of QGP with partons having already thermal masses. There are different versions of quasiparticle model such as (a) with constant parton masses (m_G or m_q) and bag constant, B [9], (b) with temperature dependent parton masses ($m_G(T)$ or $m_q(T)$) and bag constant, $B(T)$, [10, 11], (c) with further additional function called effective degrees of freedom, $D(T)$ or $\nu_g(T)$ to take account of the changes in degrees of freedom near T_c [10, 12], etc. All of them claim to explain lattice results, either by adjusting free parameters in the model or by taking lattice data on one of the thermodynamic quantity as an input and predicting other thermodynamic quantities. However, physical picture of quasiparticle model and the origin of various temperature dependent quantities are not clear yet [13]. In sQGP

[14], one considers all possible hadrons even at $T > T_c$ and try to explain non-ideal behavior of QGP near T_c .

Here we propose that the QGP near T_c is in fact what is called strongly coupled plasma [15], widely studied in QED plasma. By definition, plasma is a quasineutral gas of charged and neutral particles which exhibits collective behavior. At sufficiently high temperature neutral particles will be negligibly small so that one can see collective effects of plasma. Otherwise it will be just ordinary neutral gas and not plasma. SCP [16] is a plasma where the plasma parameter, Γ , defined as the ratio of average potential energy to average kinetic energy of the particles, is of the order of 1 or larger. Similarly, QGP is a quasi-color-neutral gas of colored particles like quarks and gluons which exhibits collective behavior. Here also to see collective effects color neutral objects like hadrons and glue balls must be negligibly small in number. Otherwise, it is just a hadron gas and not QGP. In QED plasma, recombination and ionization are taking place with the same rate. Similarly, in QGP such a process may be less because QGP is a deconfinement state where confinement effect due to QCD vacuum may be melted. Atom is a bound state, but hadron is a confinement state. Of course, there will be color Coulombic interactions due to one gluon exchange with proper quantum effects like running coupling constant etc. and may just modify the properties of QGP like EoS etc. In QED plasma also particles are interacting via Coulombic force and it modifies EoS. Just like in QED plasma, here also we can have SCP or SCQGP whenever Γ of QGP is of the order of 1 or greater. Interesting thing is here also interactions potential is Coulombic with proper modifications to take account of color degrees of freedom and quantum effects. Therefore, EoS of QGP may not much different from that of QED plasma. Only difference is in Γ . SCP is widely studied by theoretical and numerical methods and EoS of SCP is parametrized as function of Γ [16]. We will show here that the same EoS with Γ for QGP, very nicely fits the lattice results on EoS of gluon [4], 2-flavor as well as 3-flavor QGP [17]. Plasma parameters, Γ , are different for three different systems. Further extension of the model to include finite mass of the quarks, again, very nicely fits the lattice data on (2+1)-flavors and 4-flavors QGP.

2 Phenomenological Model:

In Ref. [6], we attempted to explain the non-ideal behaviour seen in lattice by considering Cornell potential, Coulomb + linear confining potential, using Mayer's cluster expansion method. It is only partially successful and near T_c , the fit to lattice data is not good. This is because Mayer's cluster expansion method is for weakly coupled system and hence concluded that QGP near T_c might be strongly coupled. In Ref. [15], we treated QGP as a strongly coupled Color-Coulombic plasma and modified EoS of SCP for gluon plasma and obtained a remarkably good fit to lattice data. Hence we confirmed that QGP near and above T_c is SCQGP and speculated that it might have dramatic effects in URHICs observables. Here we reconfirm it by fitting lattice data for all systems, namely, gluon plasma, 2-flavor and 3-flavor QGP successfully.

Let us briefly discuss the strongly coupled plasma. An extensive study of SCP [16], theoretical and partially numerical, obtained an expression for EoS of SCP as a function of Γ and is given by,

$$\varepsilon_{QED} = (3/2 + u_{ex}(\Gamma)) n T , \quad (1)$$

where the nonideal (or excess) contribution to EoS, $u_{ex}(\Gamma)$, is given by,

$$u_{ex}(\Gamma) = \frac{u_{ex}^{Abe}(\Gamma) + 3 \times 10^3 \Gamma^{5.7} u_{ex}^{OCP}(\Gamma)}{1 + 3 \times 10^3 \Gamma^{5.7}} . \quad (2)$$

Further u_{ex}^{Abe} and u_{ex}^{OCP} are given by

$$u_{ex}^{Abe}(\Gamma) = -\frac{\sqrt{3}}{2} \Gamma^{3/2} - 3 \Gamma^3 \left[\frac{3}{8} \ln(3\Gamma) + \frac{\gamma}{2} - \frac{1}{3} \right] , \quad (3)$$

$$u_{ex}^{OCP} = -0.898004 \Gamma + 0.96786 \Gamma^{1/4} + 0.220703 \Gamma^{-1/4} - 0.86097 . \quad (4)$$

u_{ex}^{Abe} was derived by Abe [18] exactly in the giant cluster-expansion theory and is valid for $\Gamma < .1$. $\gamma = 0.57721\dots$ is Euler's constant. u_{ex}^{OCP} was evaluated by computer simulation of one component plasma (OCP), where a single species of charged particles embedded in a uniform background of neutralizing charges and is valid for $1 \leq \Gamma < 180$. $u_{ex}(\Gamma)$ is valid for all $\Gamma < 180$, including the range $.1 \leq \Gamma \leq 1$. In fact, the final modified version [19] of the

original numerical solutions of hypernetted chain equation (HNC) of Springer *et. al.*, valid for $.05 \leq \Gamma \leq 50$ [20], agrees well with u_{ex}^{OCP} with discrepancies of less than 1%. $u_{ex}(\Gamma)$ is derived for strongly coupled Coulombic plasma and rigorously verified. It may be valid for any Coulombic plasma with appropriate change in charge or the coupling constant α .

Let us now consider our model, SCQGP. We assume that hadron exists for $T < T_c$ and goes to QGP for $T > T_c$. That is, for $T > T_c$ it is the plasma of quarks and gluons and no hadrons or glue balls. But it is a strongly coupled plasma, which we call SCQGP. The plasma parameter Γ is defined as,

$$\Gamma \equiv \frac{\langle PE \rangle}{\langle KE \rangle} = \frac{\frac{4}{3} \frac{\alpha_s}{r_{av}}}{T}, \quad (5)$$

where we have taken Coulombic interaction between quarks, generally used in hadron spectroscopy. The typical value of $\alpha_s \approx 0.5$, $r_{av} \approx 1 fm$ and near the critical temperature, $T_c \approx 200 MeV$, we estimate $\Gamma \approx 2/3$. Hence QGP is a strongly coupled plasma. Later we will see that the Γ which fits lattice results is indeed of the order of 1. Compared to QED, the fine structure constant, α , is replaced by $4 \alpha_s/3$ in Coulombic interaction term. r_{av} may be estimated as $r_{av} = (3/4\pi n)^{1/3}$ and hence

$$\Gamma = \left(\frac{4\pi n}{3} \right)^{1/3} \frac{4}{3} \frac{\alpha_s}{T}, \quad (6)$$

where ' n ' is the number density.

In SCP, where one generally has high enough temperature, electrons are in continuum state of atoms, i.e, ionized state, and negligibly small amount of neutral atoms. In SCQGP also there may be negligibly small hadrons due to Coulomb binding interactions. Since it is a deconfined state, there is no confinement interactions. Note the difference between sQGP and SCQGP. In sQGP presence of various neutral and colored bound states due to Coulomb interactions give rise to non-ideal effects.

Our phenomenological model for SCQGP is obtained by modifying Eq. (1), to include relativistic and quantum effects as,

$$\varepsilon = (2.7 + u_{ex}(\Gamma)) n T, \quad (7)$$

where the first term $2.7 n T$ corresponds to the ideal EOS which, in our case, may be written as $\varepsilon_s \equiv 3a_f T^4$, ideal EoS for massless relativistic gas. $a_f \equiv (16 + 21 n_f/2)\pi^2/90$ is a constant which depends on degrees of freedom. Hence we write,

$$e(\Gamma) \equiv \frac{\varepsilon}{\varepsilon_s} = 1 + \frac{1}{2.7} u_{ex}(\Gamma) . \quad (8)$$

We assume that the functional form of $u_{ex}(\Gamma)$ is the same as that of SCP, but Γ , which follows from Eq. (6), is,

$$\Gamma \equiv \left(\frac{4.4 \pi a_f}{3} \right)^{1/3} g_c \alpha_s(T) , \quad (9)$$

where we have taken $n \approx 1.1 a_f T^3$. This is motivated by the fact that SCQGP is also governed by Coulombic type interactions, i.e, Color Coulombic interaction. Only new thing is g_c and $\alpha_s(T)$. In SCP g_c is 1 and in SCQGP it is different for gluon plasma and flavored QGP because of different group structures involved in their interactions. Similarly, $\alpha_s = 1/137 = \text{constant}$ in SCP, but in SCQGP, including quantum effects, it is a running coupling constant $\alpha_s(T)$, given by,

$$\alpha_s(T) = \frac{6\pi}{(33 - 2n_f) \ln(T/\Lambda_T)} \left(1 - \frac{3(153 - 19n_f)}{(33 - 2n_f)^2} \frac{\ln(2 \ln(T/\Lambda_T))}{\ln(T/\Lambda_T)} \right) , \quad (10)$$

where n_f is the number of flavors and Λ_T is a parameter, related to QCD scale parameter. Since lattice results are with two-loop order running coupling constant, we have chosen similar form.

Finally, we have a phenomenological EoS for SCQGP with two parameters g_c and Λ_T . Both of them depends on type of QGP. Any shortcoming may be reflected in the correctness of these parameters. From Eq. (8), we get $\varepsilon(T)$ and using the relation, $\varepsilon = T \frac{\partial P}{\partial T} - P$, we get the pressure

$$\frac{P}{T^4} = \left(\frac{P_0}{T_0} + 3a_f \int_{T_0}^T d\tau \tau^2 e(\Gamma(\tau)) \right) / T^3 , \quad (11)$$

where P_0 is the pressure at some reference temperature T_0 . and may be fixed to one of the lattice data points or at critical temperature T_c . Once we know P and ε , $c_s^2 = \frac{\partial P}{\partial \varepsilon}$ can be evaluated.

In the case of QGP with finite quark masses, the n_f in the expression for a_f and hence in Γ must be replaced by effective n_f , n_f^{eff} as discussed in [17, 21].

3 Results :

In Fig. 1, we plotted $P(T)/T^4$ Vs T for pure gauge, 2-flavor and 3-flavor QGP along with lattice results. Note that, in the case of flavored QGP, since there is $(10\% \pm 5\%)$ uncertainty in P data [21] on taking continuum limit with massless quarks [21], we multiply the lattice data by the factor 1.1 and is plotted. For each system g_c and Λ_T are adjusted so that we get a good fit to lattice results. We have fixed P_0 from the lattice data at the critical temperature T_c for each system. Surprisingly good fit is obtained for all systems with $g_c = 1.4$ for gluon plasma and $g_c = 0.89$ for both 2-flavor and 3-flavor QGP. Λ_T are different for all systems with values 137.5, 80.5 and 37 for gluon plasma, 2-flavor and 3-flavor QGP respectively. We have taken n_f equal to 0, 2 and 3 respectively for three systems. The values of g_c are not that unreasonable since they are close to the eigen values of quadratic Casimir operators, 3 for pure gauge and $4/3$ for quarks.

Once $P(T)$ is obtained, then other macroscopic quantities such as ε , c_s^2 etc. are derivable from $P(T)$ and no other parameters are needed. In Fig. 2, we plotted ε/T^4 Vs T/T_c for all three systems along with lattice results and a reasonably good fit is obtained without any extra parameters. All the three curves looks similar, but shifts to left as flavor content increases. We have taken T_c equal to 275, 175 and 155 MeV respectively for gluon plasma, 2-flavor and 3-flavor QGP.

In Fig. 3, c_s^2 is plotted for all three systems, again with lattice results for gluon plasma. Reasonably good fit for gluon plasma and our predictions for the flavored QGP. All the three curves have similar behaviour, i.e, sharp rise near T_c and then flatten to the value close to $1/3$. c_s^2 is larger for larger flavor content. Very close to $T = T_c$, fits or predictions of our model may not be good, especially for ε and c_s^2 which strongly depends on variations of P with respect to T . Lattice data also has large error bars very close to T_c . However, except for small region at $T = T_c$, our results are very good for all regions of $T > T_c$.

In Figs 4 and 5, we plotted α_s and Γ respectively as a function of T/T_c . We see that QGP near $T > T_c$ upto several T_c is really strongly coupled since Γ is the order of 1. We see from the plot that pure gauge is more strongly coupled than flavored QGP. It is interesting

to note that recently Peshier and Cassing [22] also obtained similar results on Γ as a function of T in quai-particle model and concluded that QGP behaves like a liquid.

In Fig.6, we included the results on (2+1)-flavors and 4-flavors QGP from our model along with lattice data [21, 23] and replotted $P(T)/T^4$ Vs T/T_c for all systems. Similar plots for $\varepsilon(T)/T^4$ Vs T/T_c for all systems is replotted in Fig. 7. The fitted parameters for various systems are tabulated in Table 1. The effective number of flavors, n_f^{eff} is $(2 \pm 0.9672 + 0.8275)$ for (2+1)-flavor QGP and 4 ± 0.9672 , 4 ± 0.9915 for 4-flavor QGP with mass m/T equal to 0.4 and 0.2 respectively. Masses m/T in the case of (2+1)-flavors are 0.4 and 1.0 for light and heavy quarks respectively.

4 Conclusions :

Using a phenomenological model to treat QGP near and above $T = T_c$ as SCQGP, obtained by appropriate modifications of the results of SCP to take account of color and flavor degrees of freedom and quantum effects, a surprisingly good fits to lattice results are obtained. Basic idea is that SCP and SCQGP both are nonideal because of Coulombic interactions. So we expect similar EoS. Modification of SCP to SCQGP introduces system dependent two parameters which we vary to get good fit to lattice results for pressure P . ε , c_s^2 etc. then follows from P without any extra parameters. When we extend our model to include systems with finite quark mass, again, a remarkable good fits are obtained to (2+1)-flavors and 4-flavors QGP. Hence we have a unified model with just two system dependent parameters, namely SCQGP, to explain the non-ideal effects seen in the lattice simulation of QCD in various systems like pure gauge, 2-flavors, 3-flavors, (2+1)-flavors and 4-flavors QGP . Of course, to understand the values of the parameters, g_c and Λ_T , one need much more general analytic theory based on QCD. It is also interesting to see the modifications of URHICs results by treating QGP as SCQGP, since the energy density and hence temperature is in the range of few T_c .

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Table 1

Parameters of our model for various systems to get the best fit to lattice data.

| systems | $t_0 \equiv \Lambda_T/T_c$ | g_c |
|------------------------|----------------------------|-------|
| pure gauge | 0.5 | 1.4 |
| 2-flavor | 0.46 | 0.89 |
| 3-flavor | 0.24 | 0.89 |
| (2+1)-flavor | 0.24 | 1.3 |
| 4-flavor($m/T = .4$) | 0.55 | 0.36 |
| 4-flavor($m/T = .2$) | 0.55 | 0.44 |

Figure 1: Plots of P/T^4 as a function of T from our model and lattice results (symbols) for pure gauge (lower curve), 2-flavor QGP (middle curve) and 3-flavor QGP (upper curve).

Figure 2: Plots of ε/T^4 as a function of T/T_c from our model for pure gauge (lower curve), 2-flavor QGP (middle curve) and 3-flavor QGP (upper curve) and also with lattice data for pure gauge.

Figure 3: Plots of c_s^2 as a function of T/T_c from our model for pure gauge (lower curve), 2-flavor QGP (middle curve) and 3-flavor QGP (upper curve) and also with lattice data for pure gauge.

Figure 4: Plots of α_s as a function of T/T_c from our model for pure gauge (upper curve), 2-flavor QGP (middle curve) and 3-flavor QGP (lower curve).

Figure 5: Plots of Γ as a function of T/T_c from our model for pure gauge (upper curve), 2-flavor QGP (middle curve) and 3-flavor QGP (lower curve).

Figure 6: Plots of P/T^4 as a function of T/T_c from our model and lattice results (symbols) for pure gauge, 2-flavor QGP, (2+1)-flavor QGP, 3-flavor QGP and 4-flavor QGP (with two different masses).

Figure 7: Plots of ε/T^4 as a function of T/T_c from our model and lattice results (symbols) for pure gauge, 2-flavor QGP, (2+1)-flavor QGP, 3-flavor QGP and 4-flavor QGP (with two different masses).

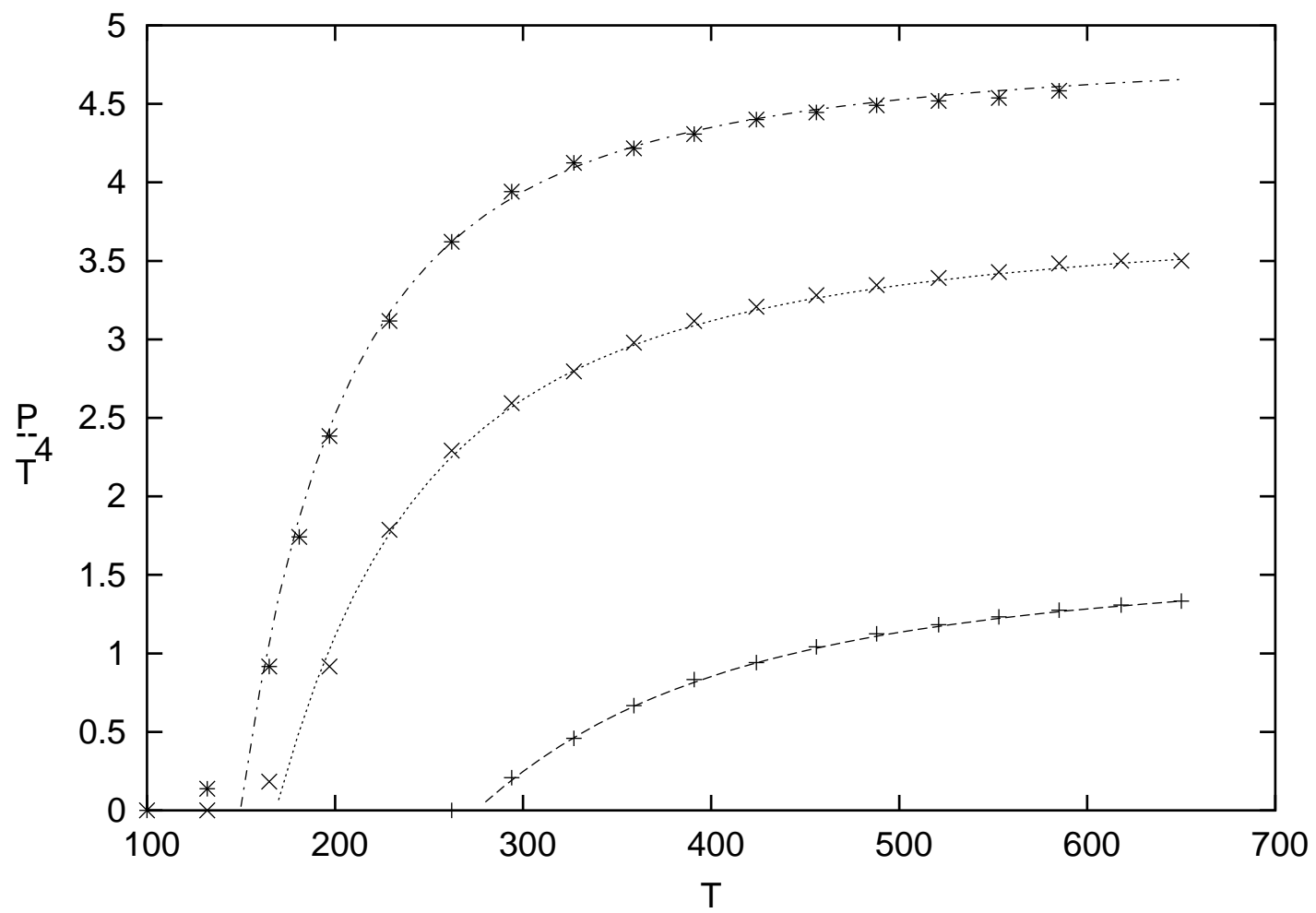


Fig. 1

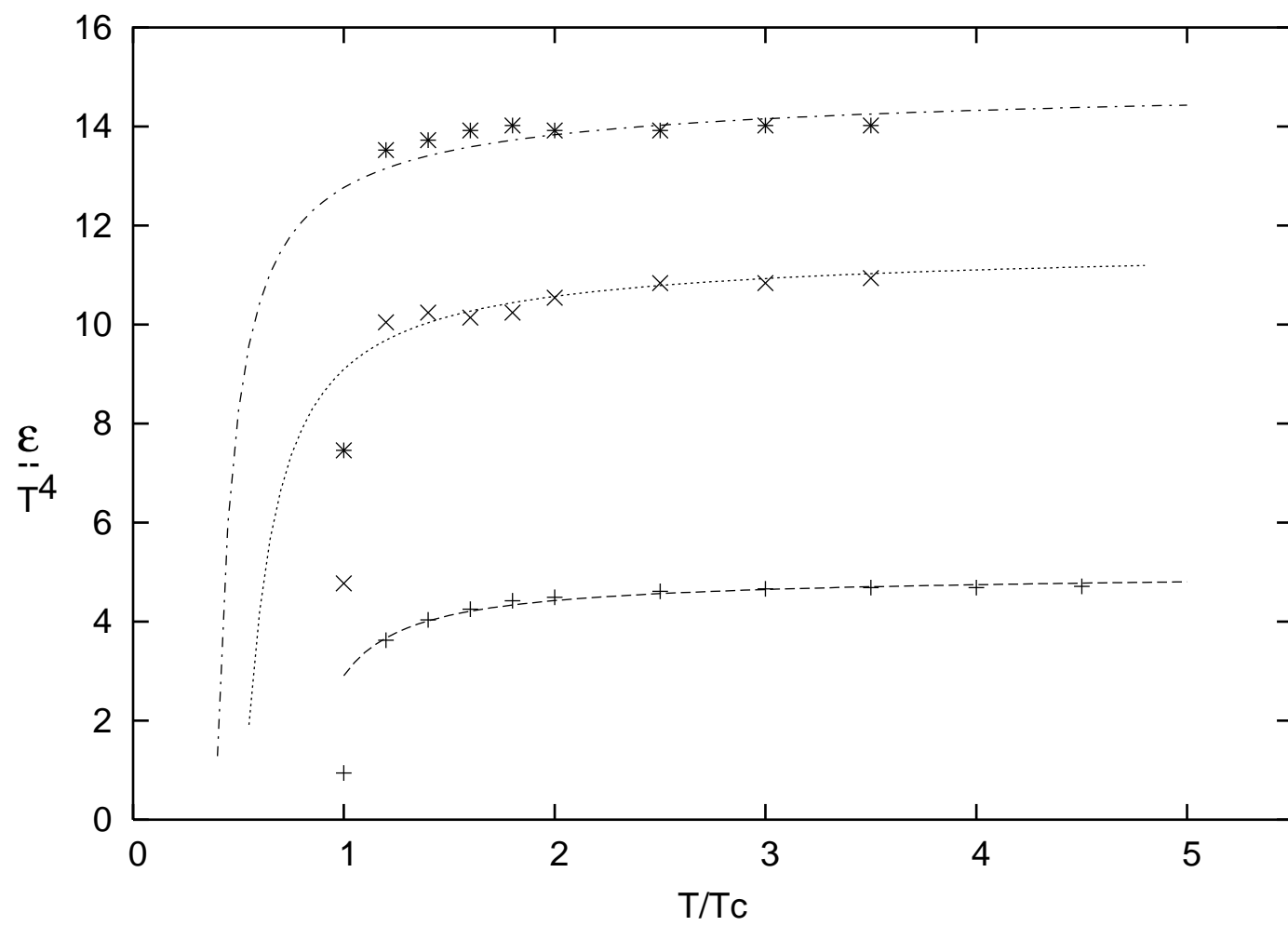


Fig. 2

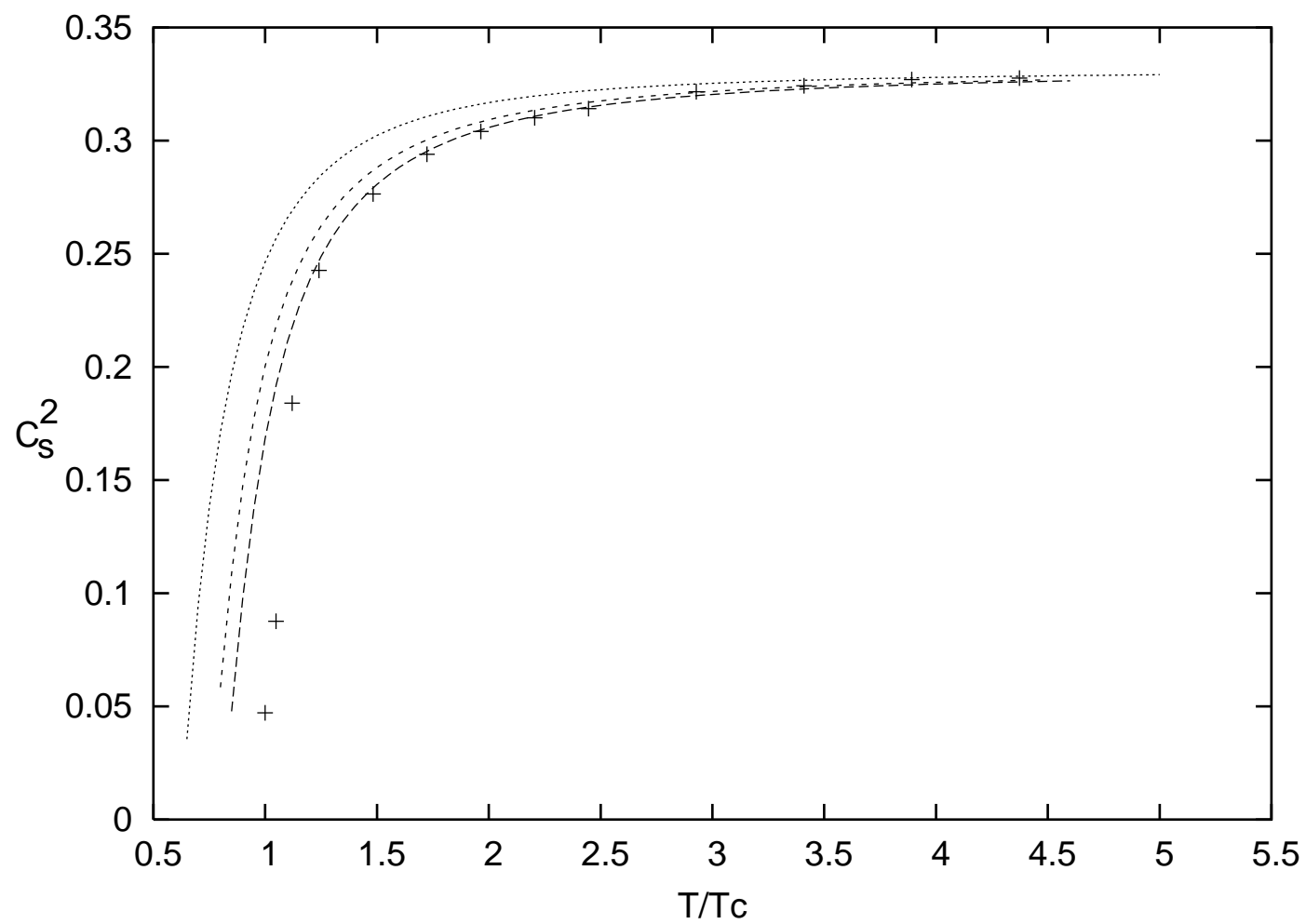


Fig. 3

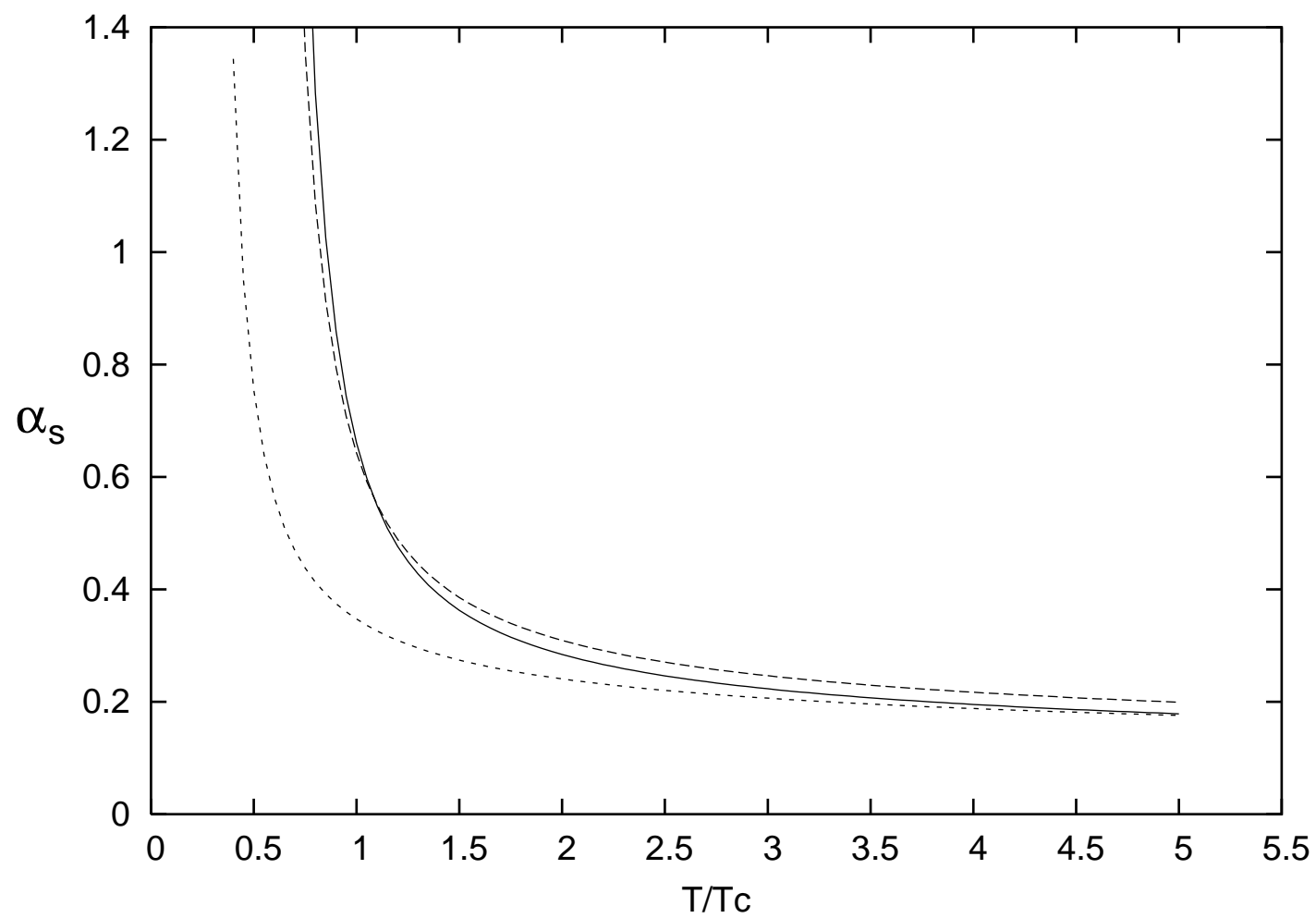


Fig. 4

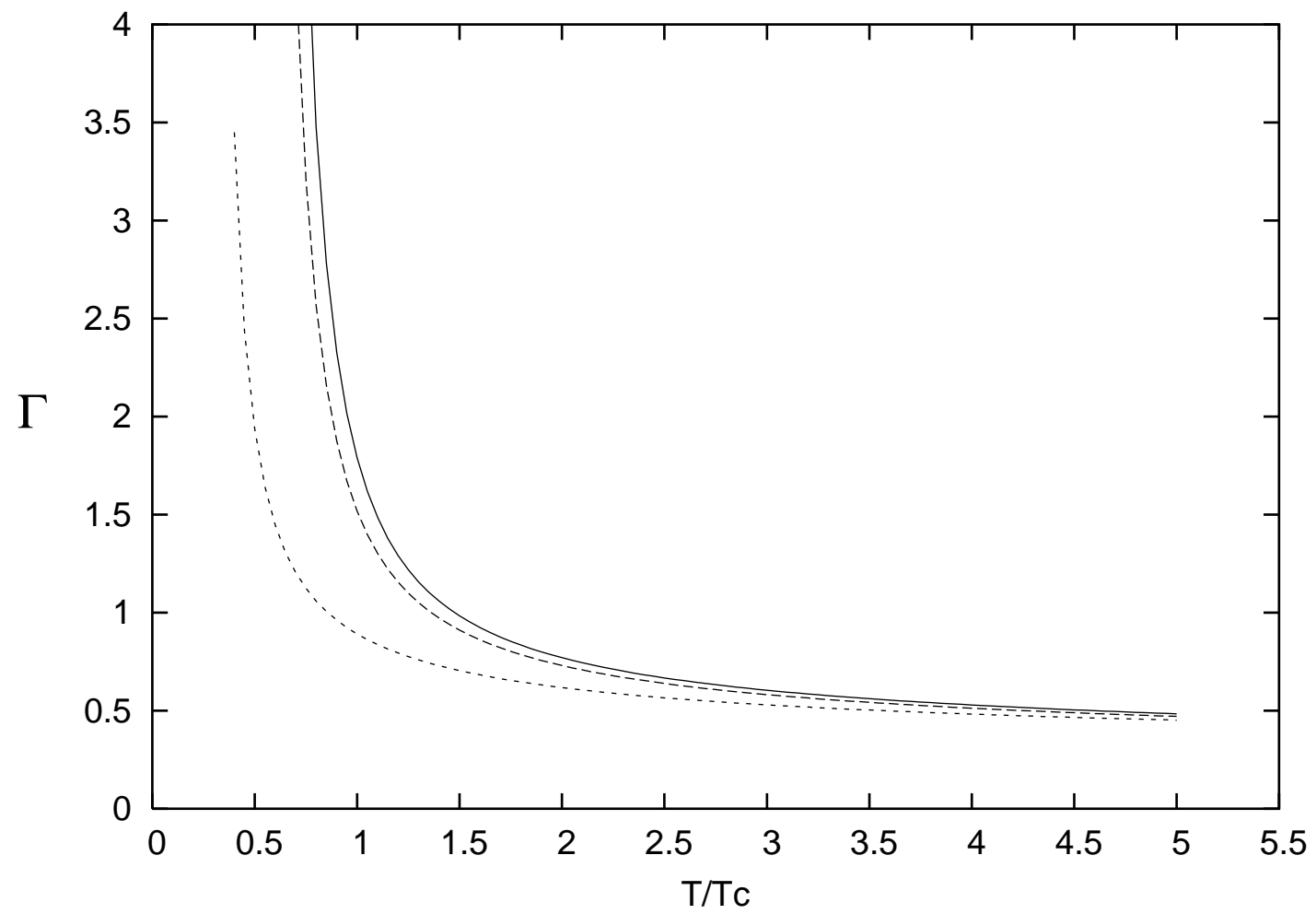


Fig. 5

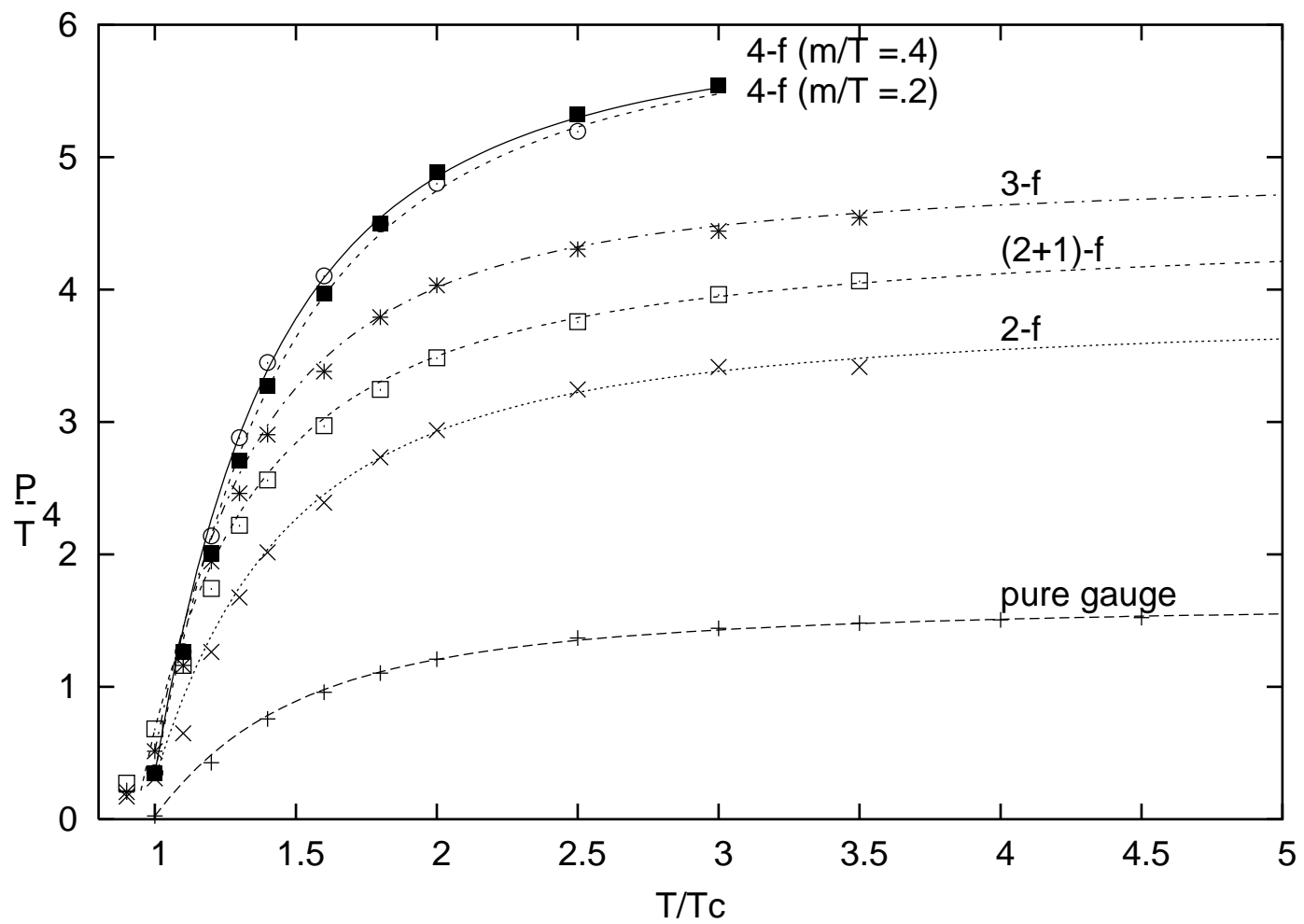


Fig. 6

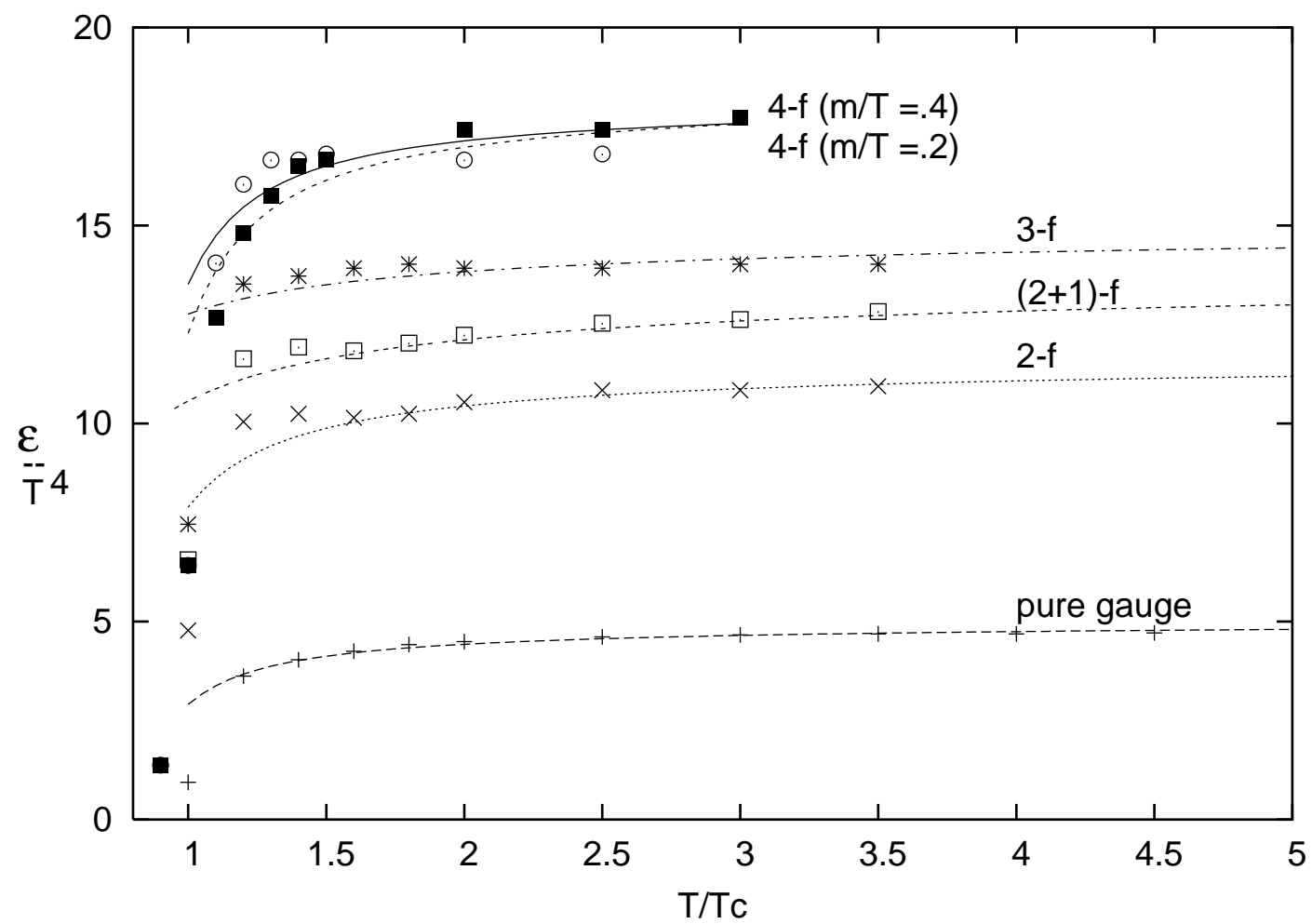


Fig. 7